# Line Charge Theory

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A consistent observation across all tribo-luminescent x-ray sources is that the electric charge on the membrane cannot be uniformly distributed across the contacted region of the membrane. When the formula for Voltage as a function of distance for a uniformly charged disc is compared with experimental measurements how the energy scales with increasing target distance in a tribo-luminescent x-ray source, the data consistently shows that the energy does not increase with distance at the rate that a uniform charge distribution would predict (see figure 1).

Figure 1:



**Energy vs Separation** 

This graph shows the energy of the band system with varying target separation distance. This data was taken from various runs on Caroline, and includes data points from a variety of loop widths. It can be seen that the energy does not increase with distance as rapidly as a unifrom charge distribution would predict.

Because the energy of a uniformly charged membrane with scales linearly with the size (assuming a constant membrane diameter to target distance aspect ration) it would be trivial to a high energy (~200 KeV) source by simply scaling the geometry of the system. Hence the inhomogeneity in the charge distribution of the membrane represents a persistent obstacle toward increasing the energy of a tribo-luminescent x-ray source. In the previous wok, this non-uniformity has been attributed to charge patches on the membrane caused by some mechanical or material variability between the membrane and its interaction with a contact material (Charge localization of a polymer surface measured by triboelectrically induced x-ray

emission, Collines et all, 2013). In this report, the case against this theory will be presented along with an alternative explanation of the non-uniformity in charge distribution. This new theory has several advantages from a scientific standpoint, and computational modelling shows that it is in good agreement with empirical measurements. Furthermore, this theory offers several insights into potential design optimizations for the future.

## Issues with a charge patch theory:

One of the main motivation for postulating a patch theory of charging has been that the contact area of older systems was uneven, or that the materials exhibited irregularities from their manufacturer or from abrasion. On the current x-ray systems, we have demonstrated through contact studies that the contact area and material properties of the membrane are nearly uniform on the 1-2mm scale, while the charge patches are hypothesized to be on the order of 0.5-1mm in diameter.

Likewise, the possibility of variations in the membrane material resulting in charge patches also contradicts the wide body of literature on tribo-charging which demonstrates roughly uniform charging on membranes charged at atmospheric pressure. Furthermore, there is no physical explanation for why the anisotropies in charge do not homogenate over time through conduction. Because a charge patch is at a higher energy density than its surroundings, its charge should flow outwards over time – and thus the charge on the membrane should gradually become uniform.

Finally, and most crucially, any theory of triboluminescence must account for the fact that the presence of light is the result of electrical discharge, and this in turn implies that the system ought to be modelled dynamically.

## Line Charge Theory:

Since triboluminescence is the result of an electric discharge from a charged membrane, we know that the charge on the membrane is to some extent a function of the discharge current. Experiments have shown that the discharge current can be around 50% of the estimated membrane charge - suggesting that the loss of charge from the membrane is significant. Although the mechanism and distribution of this discharge are not well understood, we shall attempt to use this insight to model the electrical behavior of a triboluminescent x-ray source. According to this hypothesis we posit that after separation from the contact, the electric field on the membrane causes it to quickly lose most of its charge within a small area. Since the rate of the discharge is not well understood, we will use a highly simplified approximation, and model the charge on the membrane as being localized on a shelf of uniform charge density, extending out some distance x past point of separation, that then drops to zero. (see figure 2)



This theory has the conceptual advantage that it can explain the non-uniformity of charge on the membrane through a process which is known to occur in the generation of tribogenic x-rays – namely the discharging of the membrane itself to create x-rays. Furthermore, this hypothesis facilitates a theoretical prediction of the relationship between the energy and separation of the band which conforms to the empirical findings.

# **Empirical and Computational Testing:**

A computational program to calculate the electrostatic behavior of the system described above was created in Matlab. This model assumed that discharge of electrons occurred at the shelf in the charge, and numerically calculated the mean energy of electrons discharged from this point onto a large conducting target separated by some distance d. The extent of the charge patch from the point of separation was varied in order to establish some estimate of how localized the decay process might be.

A similar physical experiment was run, by measure the energy of the electrical discharge of a triboluminescent membrane as a function of the separation distance of a target anode.

A comparison of the results of this experiment is shown, along with the best fit model of the computational simulation in figure 3.





This graph shows the energy of the band system with varying target separation distance. This data was taken from various runs on Caroline, and includes data points from a variety of loop widths. These measurements fit the theoretical curve of a 2mm long rectangular charged region. (2mm was determined to be the best fit through least squares methods).

It can be seen in the above graph that the theoretical relationship between energy and separation for a thin line of charge closely matches the observed relationship. A least squares method has shown that a charged rectangle extending 2mm beyond the point of separation charge provides the closest match to the data.

### **Results and Discussion:**

It should also be noted that this model of the electric field can be used to extrapolate a charge density of the line charge region. For a 2mm long charge rectangle to produce 36 KeV electrons at a separation of 3mm, it would need a charge density of 4.7e-4 C/m^2, which corresponds to 3e11 electrons/cm^2. This figure can then be used to set a theoretical upper bound on the x-ray flux of such a system. If we assume that this entire charge of a charged area of membrane is discharged to produce x-rays with the measured efficiency for a Pb target (6000 electrons/x-ray), then we find that the maximum x-ray production efficiency is 5e7 x-rays/cm^2. Using this we can then set lower bound on the membrane feed rate required to generate a flux of e12 x-rays/sec at 2e4cm^2/sec. It should be noted that this could be achieved using a 10cm wide band rotating at 20m/s.

Beyond successfully modeling the observed relationship between energy and target separation distance, the line charge hypothesis also implies that increasing the width of the band provides only marginal increases in the energy for widths beyond 1/2" (see figure 4). Figure 4:



This finding has, to some extent, been borne out in experimental observations with the band up to one inch. Moreover, this graph implies a fundamental limitation with the current incarnation of the band system, since it shows that scaling up the dimensions of the band has very little effect on the energy of the discharge. If we assume a 1mm long charge region, then it would require a 15cm wide band to get 100 KeV. Furthermore, in order to get 150 KeV, it would require a band 1.5 meters wide.

As the above results implies, raising the energy of the band to the levels required for x-ray imaging will require a some change in the performance of the system. According to the terms of the line charge theory, the most effective way to increase the energy of a system is to increase the spatial persistence of the charge region past the point of separation. (see figures 4 & 5).

#### Figure 5:



The preceding two graphs suggest that the energy of the band system can be increased by scaling up the system, provided that the length of the charging region past the separation point is increased. Presently, it is unclear how to achieve this feat, though possible solutions might include changes in the electrical properties of the membrane material such as lowering the conductivity or reducing losses and unwanted discharges from the membrane

### **Conclusions:**

The results of this report can be summarized as follows:

- Modeling the charge distribution of the band as thin rectangle extending across the width of the band and outward from the point of separation yields a good first pass theoretical description of the energy.
- This hypothesis explains the limitations in the gain in energy that can be achieved through increasing the width and height of the discharge region, and can also help explain the failure of band folding to increase energy.
- Given this theory, it seems unlikely that the current incarnation of the band can achieve energies beyond than 100KeV by simply increasing its size.
- The simplest way to increase the energy of the band is to extend the charged region further outward from the separation point.